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PREFACE

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International E-Conference on Differential Equations and Applications

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The Special Issue ICMDS-IE-CDEA 2021 is dedicated to researchers working in the fields of qualitative analysis and numerical simulation of linear and nonlinear mathematical models.

The papers presented in this special are selected from thirty eight documents presented in two international conferences that took place in 2021. The first is ICMDS-2021 (International Conference on Mathematics & Data Science 2 Khouribga, Morocco, 28-30 October, 2021). The second is IE-CDEA-2021 (International E-Conference on Differential Equations and Applications, LAMA, Fez, Morocco, 24-25 September 2021).

Each submission to this special issue has gone through a thorough and rigorous reviewing process, in accordance with the high standard of the *Advanced Mathematical Models & Applications (AMMA).* We thank the reviewers for their assiduous and meticulous reviewing, which contributed to the high quality of the papers.

With a stringent peer review process, there are thirteen papers included in this Special Issue, which cover the following aspects within the context of linear and nonlinear mathematical models: 1) Mathematical analysis of differential equations; 2) Analysis of problems with nonlinear PDE; 3) Numerical simulation. A brief summary of the accepted papers is discussed in the following.

1. Mathematical analysis of differential equations

   Fractional calculus attracts the attention of many researchers due to their applications in modeling of different phenomena in physics, engineering, optimization, data science, biology, finance, chemistry and image processing Aboud et al. (2012); Jun & Zhihui (2011); Nachaoui & Laghrib (2022) and the references therein. The first contribution in this field is in Bouaouid et al. (2023) where the authors investigated mild solutions for the Cauchy problem presented in Atraoui & Bouaouid (2023) with an integral term. Under certain conditions on the data, they show the existence and uniqueness of the solution of this Cauchy problem using the Schaefer fixed point theorem combined with the cosine family of linear operators.

   The controllability is a most important qualitative behavior of a dynamical system. The contribution of Atraoui et al. (2023) is in this field. The authors elaborated a qualitative study of the controllability of mild solutions for a nonlocal fractional conformable Cauchy problem of differential equations of the second order. After recalling some tools related to the conformable fractional calculus, they present their main result. This is essentially based on the conformable fractional Laplace transform and the Banach fixed point theorem.

Furthermore, an optimal control problem for variable-order differential system is considered by the authors in Karite & Khazari (2023). The existence and uniqueness of the solution is proved for this type of systems and the optimal control is computed. The optimal problem is a generalization of the one of parabolic systems with Dirichlet conditions considered by Lions (1988). The work done here could be extended to the case of semilinear fractional systems as well as to systems expressed with other types of fractional derivatives.

2. Analysis of problems with nonlinear PDE

Authors of El Hammar et al. (2023) devoted their attention to the existence of non-trivial weak solutions for nonlocal (Kirchhoff type) parabolic equations. The proof is based on a transformation of this nonlocal parabolic problem into a new one governed by an operator equation of the form $Lu + Fu = g$, where F is a semicontinuous bounded map and L is a linear densely defined maximal monotone operator in an appropriate space.

Motivated by physical models, the authors in Hmidouch et al. (2023) have carried out the study of the weak solutions of a nonlinear degenerate parabolic problem with Dirichlet type boundary conditions. To prove the existence and uniqueness of the weak solutions, they use a variational method with a semi-discretization of this equation.

In El Ahmadi et al. (2023) the authors are interested in the existence and multiplicity of nontrivial weak solutions for a class of double phase problem with variable exponents, where the nonlinearity is superlinear but does not satisfy the $(\mathbf{AR})-$ condition. The proofs rely on variational arguments based on the Mountain Pass Theorem and the Fountain Theorem with Cerami condition. The main novelty comes from the fact that the problem considered, has a more complicated structure, is modeled in a non-classical Sobolev space, requiring more delicate and complicated estimates to handle the nonlinear boundary condition.

The main goal of the paper Ait Temghart et al. (2023) is to extend the results established in Chipot & de Oliveira (2019).The authors consider the existence of entropy solutions to the an elliptic (p, q)-Laplacian problem when p and q are nonlocal quantities. They get the results by assuming the right-hand side function to be an integrable function.

Motivated by an unilateral problem, authors in Hassib & Akdim (2023) extended a previous result of Brezis & Browder (1982) in the setting of the weighted variable exponent Sobolev space. They stated and proved some properties of the capacity in the setting of the weighted variable exponent Sobolev spaces. Then they applied the obtained results in the study of a unilateral problem.

The role played by function spaces with variable smoothness and integrability in some application, including the modeling of field of electronic fluid mechanics, image restoration and optimization, motivated the study developed in Ouidirne et al. (2023). The authors proved the embeddings between Besov-Morrey spaces and Triebel-Lizorkin-Morrey spaces with variable exponent. They also give Sobolev-type embeddings in Besov-Morrey spaces with variable exponents.

3. Numerical simulation

In the context of numerical simulation, the authors in Jelti et al. (2023) considered a mathematical model combining the shallow water equations for water-sediment mixture, the sediment transport diffusion and the bed morphology change equations. The system is solved by the finite volume Roe scheme, associated with a treatment of the source term developed in Jelti et al. (2017). Two problems of open channel hydraulics where considered: a channel over mobile and horizontal bed, and a channel over mobile bed with

a bump. Through the obtained results, the numerical scheme detected the bed rate change and changes on velocity, concentration as well as free water-surface profiles.

In the field of valuation of financial derivatives, a neural network approach to price the American put option under stochastic volatility has been proposed in El Kharrazi et al. (2023). The neural network to solve the Black Scholes partial differential equation Black & Scholes (1973) was combined with an explicit finite difference method using radial basis function. The comparison of the performance of the radial basis function neural network approach with the results presented in previous studies showed that the suggested approach provides a better perspective to describe the behavior of the option pricing model, and may be chosen owing to its reliability and accuracy while requiring a minimal computational effort.

In the paper Ouidirne et al. (2023), an approximate Bayesian inference which can be seen for the deterministic model as a Bayesian optimization problem that searches for the parameters of the model that best fit the observed data is adopted. The authors introduced an approximate Bayesian inference algorithm to estimate the model parameters. This algorithm was first tested on simulated data and gave satisfactory results. Then the introduced model was illustrated on Morocco COVID-19 data. Simulated datasets are used to test the efficiency of the proposed algorithm.

Last but not least, the author in Nachaoui (2023) discuss the convergence and computational efficiency of the iterative alternating method Berdawood et al. (2023); Nachaoui et al. (2021) applied to a Cauchy problem Aboud et al. (2021); Ellabib et al. (2021) governed by the convection-diffusion equation. The method is based on alternating the solutions of two different problems governed by the convection-diffusion equation. At each iteration, the method updates the boundary condition on a part of the boundary and solves the two problems sequentially until convergence. The numerical results showed that the iterative alternating method is a robust and efficient algorithm for solving this kind of Cauchy problems.

We strongly believe that this collection of research papers will benefit a larger audience and open doors for new investigations in the field of analysis and numerical simulation of linear and nonlinear mathematical models.

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